كلية الحاسبات والذكاء الإصطناعي

BS111

Probability and Statistics

Lecture 08

Dr. Ahmed Hagag

Faculty of Computers and Artificial Intelligence
Benha University

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Chapter 2: Random Variable

- Discrete Random Variables.
- Probability Mass Functions.
- Cumulative Distribution Functions.
- Discrete R.V. (Mean and Variance).
- Continuous Random Variables.
- Probability Density Functions.
- Continuous R.V. (Mean and Variance).
- Joint Probability Distributions.

Independence (1/3)

Independence:

If X and Y are two random variables, the X and Y are independent if any one of the following properties is true:

1) $f_{XY}(x,y) = f_X(x)f_Y(y)$ for all x and y

2) $f_{X|y}(x) = f_X(x)$ for all x and y with $f_Y(y) > 0$

3) $f_{Y|x}(y) = f_Y(y)$ for all x and y with $f_X(x) > 0$

Independence (2/3)

Example1:

$f_{XY}(x, y)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

Find:

$f_X(x)$

$f_Y(y)$

Are X and Y independent?

Independence (3/3)

Example1 – Answer (1/3):

$f_{XY}(x, y)$

Find:

$f_X(x)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8
$f_X(x)$	**1/4**	**3/8**	**1/4**	**1/8**

Independence (3/3)

Example1 – Answer (1/3):

$f_{XY}(x, y)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8
$f_X(x)$	1/4	3/8	1/4	1/8

Same as

Find:

$f_X(x)$

x	1	1.5	2.5	3
$f_X(x)$	1/4	3/8	1/4	1/8

Independence (3/3)

Example1 – Answer (2/3):

$f_{XY}(x, y)$

Find:

$f_Y(y)$

y \ x	1	1.5	2.5	3	$f_Y(y)$
1	1/4	0	0	0	**1/4**
2	0	1/8	0	0	**1/8**
3	0	1/4	0	0	**1/4**
4	0	0	1/4	0	**1/4**
5	0	0	0	1/8	**1/8**

Independence (3/3)

Example1 – Answer (2/3):

$f_{XY}(x,y)$

Find:

$f_Y(y)$

Same as

y	$f_Y(y)$
1	1/4
2	1/8
3	1/4
4	1/4
5	1/8

y \ x	1	1.5	2.5	3	$f_Y(y)$
1	1/4	0	0	0	1/4
2	0	1/8	0	0	1/8
3	0	1/4	0	0	1/4
4	0	0	1/4	0	1/4
5	0	0	0	1/8	1/8

Independence (3/3)

Example1 – Answer (2/3):

$f_{XY}(x, y)$

Find:

$f_Y(y)$

y	$f_Y(y)$
1	1/4
2	1/8
3	1/4
4	1/4
5	1/8

Same as

y	1	2	3	4	5
$f_Y(y)$	1/4	1/8	1/4	1/4	1/8

Independence (3/3)

Example1 – Answer (3/3):

$f_{XY}(x, y)$

Are X and Y independent?

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

Independence (3/3)

Example1 – Answer (3/3):

$f_{XY}(x, y)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

Are X and Y independent?

If:

$$f_{XY}(x, y) = f_X(x) f_Y(y) \text{ for all } x \text{ and } y$$

Independence (3/3)

Example1 – Answer (3/3):

$f_{XY}(x,y)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

Same as

x	1	1.5	1.5	2.5	3
y	1	2	3	4	5
$f_{XY}(x,y)$	**1/4**	**1/8**	**1/4**	**1/4**	**1/8**

Independence (3/3)

Example1 – Answer (3/3):

x	1	1.5	1.5	2.5	3
y	1	2	3	4	5
$f_{XY}(x,y)$	**1/4**	**1/8**	**1/4**	**1/4**	**1/8**

x	**1**	**1.5**	**2.5**	**3**
$f_X(x)$	**1/4**	**3/8**	**1/4**	**1/8**

y	**1**	**2**	**3**	**4**	**5**
$f_Y(y)$	**1/4**	**1/8**	**1/4**	**1/4**	**1/8**

$$f_{XY}(1,1) = \frac{1}{4}$$

$$f_X(1)f_Y(1) = \frac{1}{4} * \frac{1}{4} = \frac{1}{16}$$

Independence (3/3)

Example1 – Answer (3/3):

x	1	1.5	1.5	2.5	3
y	1	2	3	4	5
$f_{XY}(x,y)$	**1/4**	**1/8**	**1/4**	**1/4**	**1/8**

x	**1**	**1.5**	**2.5**	**3**
$f_X(x)$	**1/4**	**3/8**	**1/4**	**1/8**

y	**1**	**2**	**3**	**4**	**5**
$f_Y(y)$	**1/4**	**1/8**	**1/4**	**1/4**	**1/8**

$$f_{XY}(1,1) = \frac{1}{4} \neq f_X(1)f_Y(1) = \frac{1}{4} * \frac{1}{4} = \frac{1}{16}$$

Then X and Y are not independent.

Linear Relationship (1/6)

Covariance (1/5):

The covariance between the random variables X and Y is the measure of *linear relationship* between them, denoted as $cov(X,Y)$ or σ_{XY}, where

$$cov(X,Y) = \sigma_{XY} = E\Big(\big(X - E(X)\big)\big(Y - E(Y)\big)\Big)$$

$$= E(XY) - E(X)E(Y)$$

Linear Relationship (1/6)

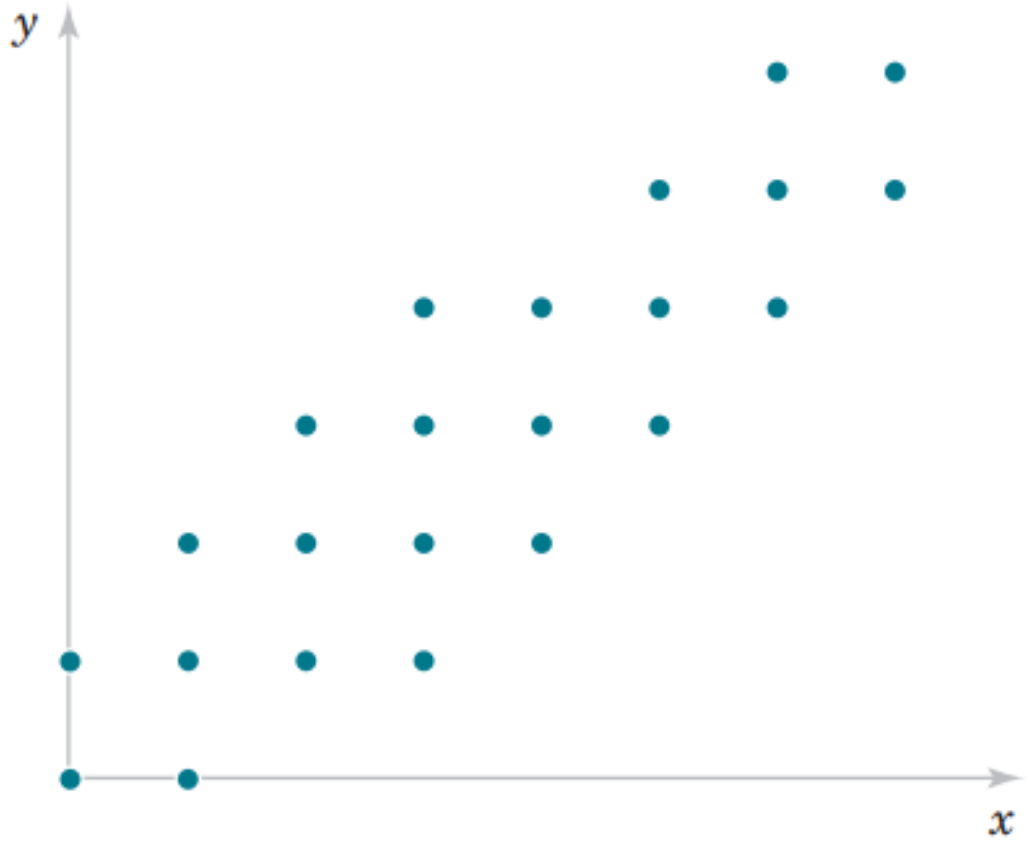
Covariance (2/5):

Covariance measures the total variation of two random variables from their expected values. Using covariance, we can only standard the direction of the relationship. However, it does not indicate the strength of the relationship.

Linear Relationship (1/6)

Covariance (3/5):

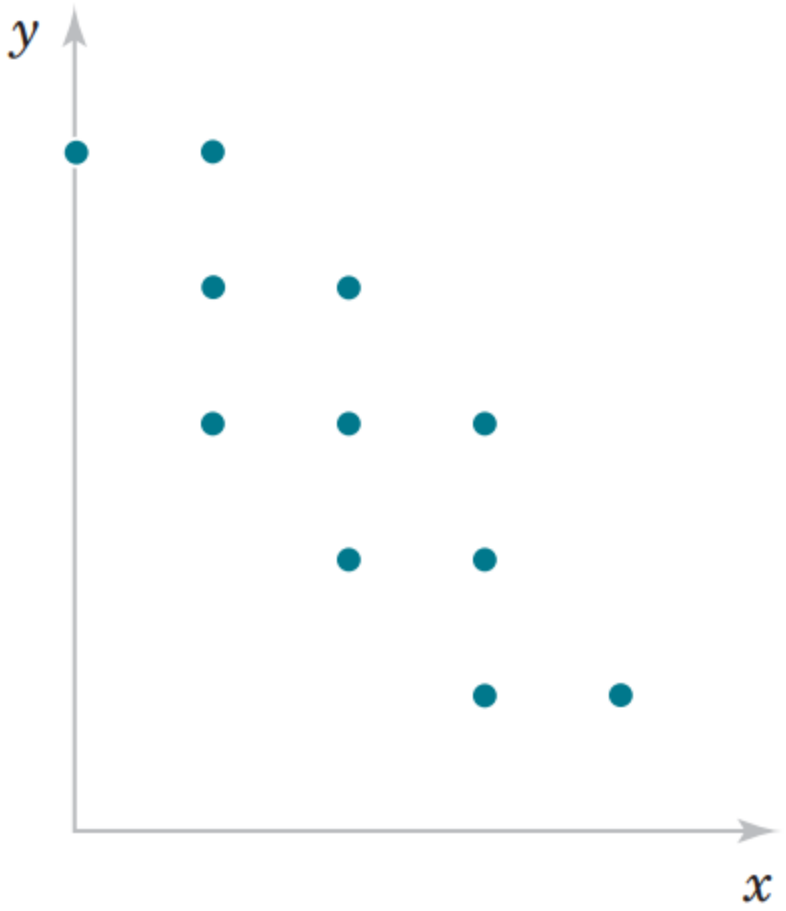
Positive covariance



Linear Relationship (1/6)

Covariance (4/5):

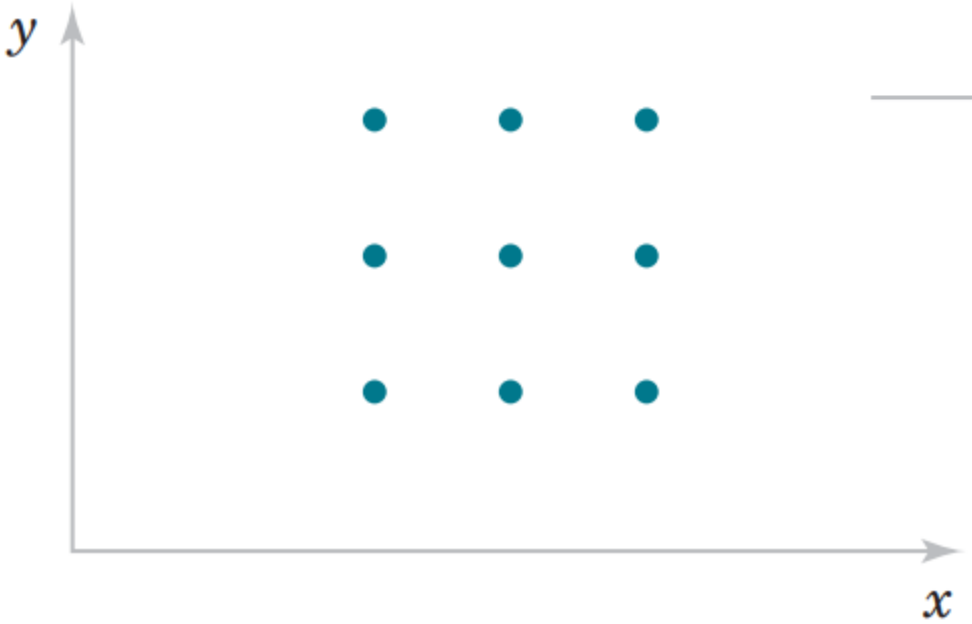
Negative covariance

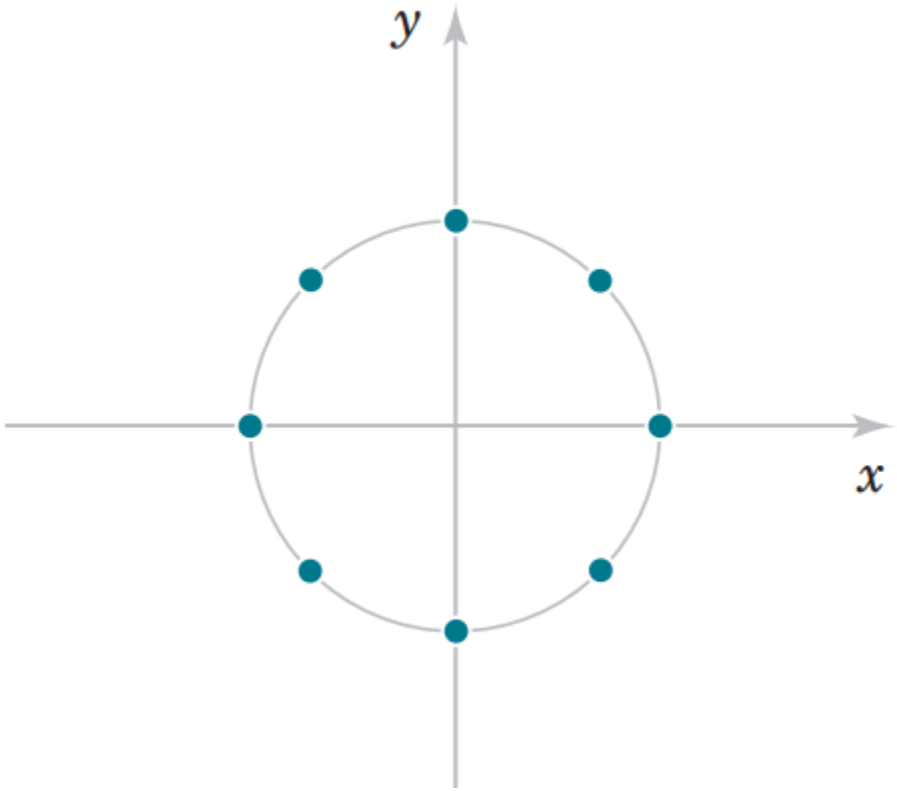


Linear Relationship (1/6)

Covariance (5/5):

Zero covariance





Linear Relationship (2/6)

Example1:

$f_{XY}(x, y)$

y \ x	1	2	4
3	1/8	0	0
4	1/4	0	0
5	0	1/2	0
6	0	0	1/8

Determine the covariance σ_{XY}?

$$cov(X, Y) = \sigma_{XY} = E(XY) - E(X)E(Y)$$

Linear Relationship (2/6)

Example1:

$f_{XY}(x, y)$

y \ x	1	2	4
3	1/8	0	0
4	1/4	0	0
5	0	1/2	0
6	0	0	1/8

Same as

x	1	1	2	4
y	3	4	5	6
$f_{XY}(x, y)$	**1/8**	**1/4**	**1/2**	**1/8**

Linear Relationship (3/6)

Example1 – Answer (1/6):

$f_{XY}(x,y)$

x	1	1	2	4
y	3	4	5	6
$f_{XY}(x,y)$	**1/8**	**1/4**	**1/2**	**1/8**

x	y	$f_{XY}(x,y)$	$x\,f_{XY}(x,y)$	$y\,f_{XY}(x,y)$	$xy\,f_{XY}(x,y)$
1	3	1/8			
1	4	1/4			
2	5	1/2			
4	6	1/8			

Linear Relationship (3/6)

Example1 – Answer (2/6):

x	y	$f_{XY}(x,y)$	$x\,f_{XY}(x,y)$	$y\,f_{XY}(x,y)$	$xy\,f_{XY}(x,y)$
1	3	1/8	1/8		
1	4	1/4	1/4		
2	5	1/2	2/2		
4	6	1/8	4/8		

Linear Relationship (3/6)

Example1 – Answer (3/6):

x	y	$f_{XY}(x,y)$	$x\,f_{XY}(x,y)$	$y\,f_{XY}(x,y)$	$xy\,f_{XY}(x,y)$
1	3	1/8	1/8	3/8	
1	4	1/4	1/4	4/4	
2	5	1/2	2/2	5/2	
4	6	1/8	4/8	6/8	

Linear Relationship (3/6)

Example1 – Answer (4/6):

x	y	$f_{XY}(x,y)$	$x\,f_{XY}(x,y)$	$y\,f_{XY}(x,y)$	$xy\,f_{XY}(x,y)$
1	3	1/8	1/8	3/8	3/8
1	4	1/4	1/4	4/4	4/4
2	5	1/2	2/2	5/2	10/2
4	6	1/8	4/8	6/8	24/8

Linear Relationship (3/6)

Example1 – Answer (5/6):

x	y	$f_{XY}(x,y)$	$x\,f_{XY}(x,y)$	$y\,f_{XY}(x,y)$	$xy\,f_{XY}(x,y)$
1	3	1/8	1/8	3/8	3/8
1	4	1/4	1/4	4/4	4/4
2	5	1/2	2/2	5/2	10/2
4	6	1/8	4/8	6/8	24/8
Sum			**15/8**	**37/8**	**75/8**
			$E(X)$	$E(Y)$	$E(XY)$

Linear Relationship (3/6)

Example1 – Answer (6/6):

$\boldsymbol{E(X) = 15/8}$ $\boldsymbol{E(Y) = 37/8}$ $\boldsymbol{E(XY) = 75/8}$

$$cov(X,Y) = \sigma_{XY} = E(XY) - E(X)E(Y)$$

$$cov(X,Y) = \sigma_{XY} = \frac{75}{8} - \frac{(15)(37)}{64} = 0.703125$$

Positive covariance

Linear Relationship (4/6)

Correlation (1/2):

The correlation between the random variables X and Y is just scales the covariance by the product of the standard deviation of each variable. Correlation measures the strength of the relationship between variables and denoted as ρ_{XY}, where

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

$$-1 \leq \rho_{XY} \leq +1$$

Linear Relationship (4/6)

Correlation (2/2):

If X and Y are independent random variables,

$$\rho_{XY} = \sigma_{XY} = 0$$

However, if the correlation between two random variables is zero, we cannot conclude that the random variables are independent.

Linear Relationship (5/6)

Example2:

$f_{XY}(x, y)$

y \ x	1	2	4
3	1/8	0	0
4	1/4	0	0
5	0	1/2	0
6	0	0	1/8

Determine the correlation ρ_{XY}?

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

Linear Relationship (6/6)

Example2 – Answer (1/4): (From the previous example)

x	y	$f_{XY}(x,y)$	$x\,f_{XY}(x,y)$	$y\,f_{XY}(x,y)$	$xy\,f_{XY}(x,y)$
1	3	1/8	1/8	3/8	3/8
1	4	1/4	1/4	4/4	4/4
2	5	1/2	2/2	5/2	10/2
4	6	1/8	4/8	6/8	24/8
Sum			**15/8**	**37/8**	**75/8**
			$E(X)$	$E(Y)$	$E(XY)$

Linear Relationship (6/6)

Example2 – Answer (2/4):

New

x	y	$f_{XY}(x,y)$	$x\,f_{XY}(x,y)$	$y\,f_{XY}(x,y)$	$xy\,f_{XY}(x,y)$	$x^2\,f_{XY}(x,y)$	$y^2\,f_{XY}(x,y)$
1	3	1/8	1/8	3/8	3/8	1/8	9/8
1	4	1/4	1/4	4/4	4/4	1/4	16/4
2	5	1/2	2/2	5/2	10/2	4/2	25/2
4	6	1/8	4/8	6/8	24/8	16/8	36/8
Sum			**15/8**	**37/8**	**75/8**	**35/8**	**177/8**
			$E(X)$	$E(Y)$	$E(XY)$	$E(X^2)$	$E(Y^2)$

Linear Relationship (6/6)

Example2 – Answer (3/4):

$E(X) = 15/8$ $E(Y) = 37/8$ $E(XY) = 75/8$

$E(X^2) = 35/8$ $E(Y^2) = 177/8$

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

$$\sigma_X = \sqrt{V(X)} = \sqrt{E(X^2) - \big(E(X)\big)^2} = \sqrt{0.859375}$$

$$\sigma_Y = \sqrt{V(Y)} = \sqrt{E(Y^2) - \big(E(Y)\big)^2} = \sqrt{0.734375}$$

$$\sigma_{XY} = E(XY) - E(X)E(Y) = 0.703125$$

Linear Relationship (6/6)

Example2 – Answer (4/4):

$E(X) = 15/8$ $E(Y) = 37/8$ $E(XY) = 75/8$

$E(X^2) = 35/8$ $E(Y^2) = 177/8$

$$\rho_{XY} = \frac{0.703125}{\sqrt{(0.859375)(0.734375)}} = 0.885079$$

Joint Prob. Density Fun. (1/19)

Joint Probability Density Function:

If X and Y are two continuous random variables, the joint probability density fun. is denoted as $f_{XY}(x,y)$, satisfies

1) $f_{XY}(x,y) \geq 0$ for all x, y

2) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\ dxdy = 1$

3) For any region R of two−dimensional space,

$$P\big((X,Y) \in R\big) = \int\int_{R} f_{XY}(x,y)\ dxdy$$

Joint Prob. Density Fun. (2/19)

Double Integration:

$$\int_0^1 \int_1^2 xy^2 \, dxdy$$

$f(x,y)$

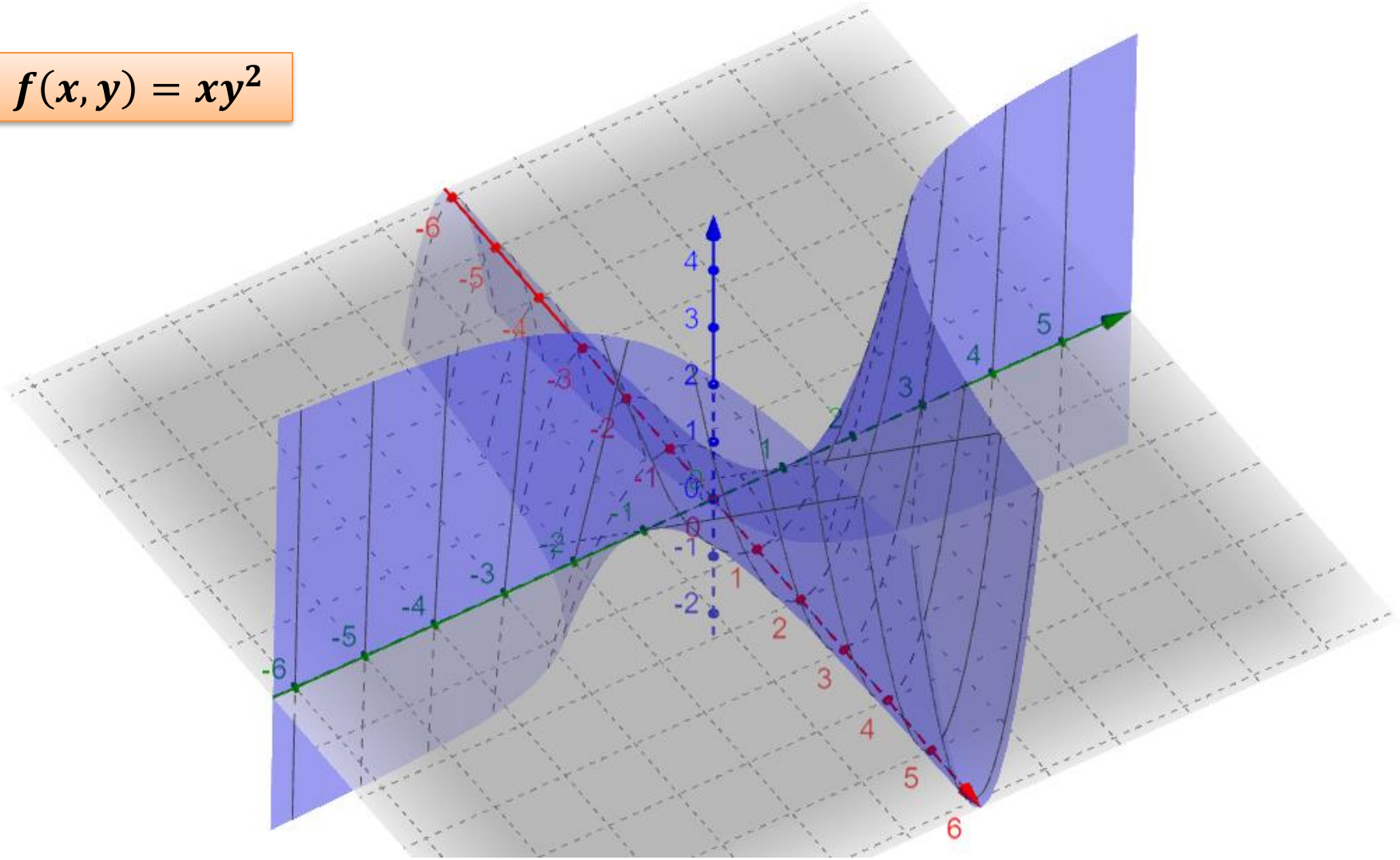
Same as:

$$\iint_R xy^2 \, dxdy$$

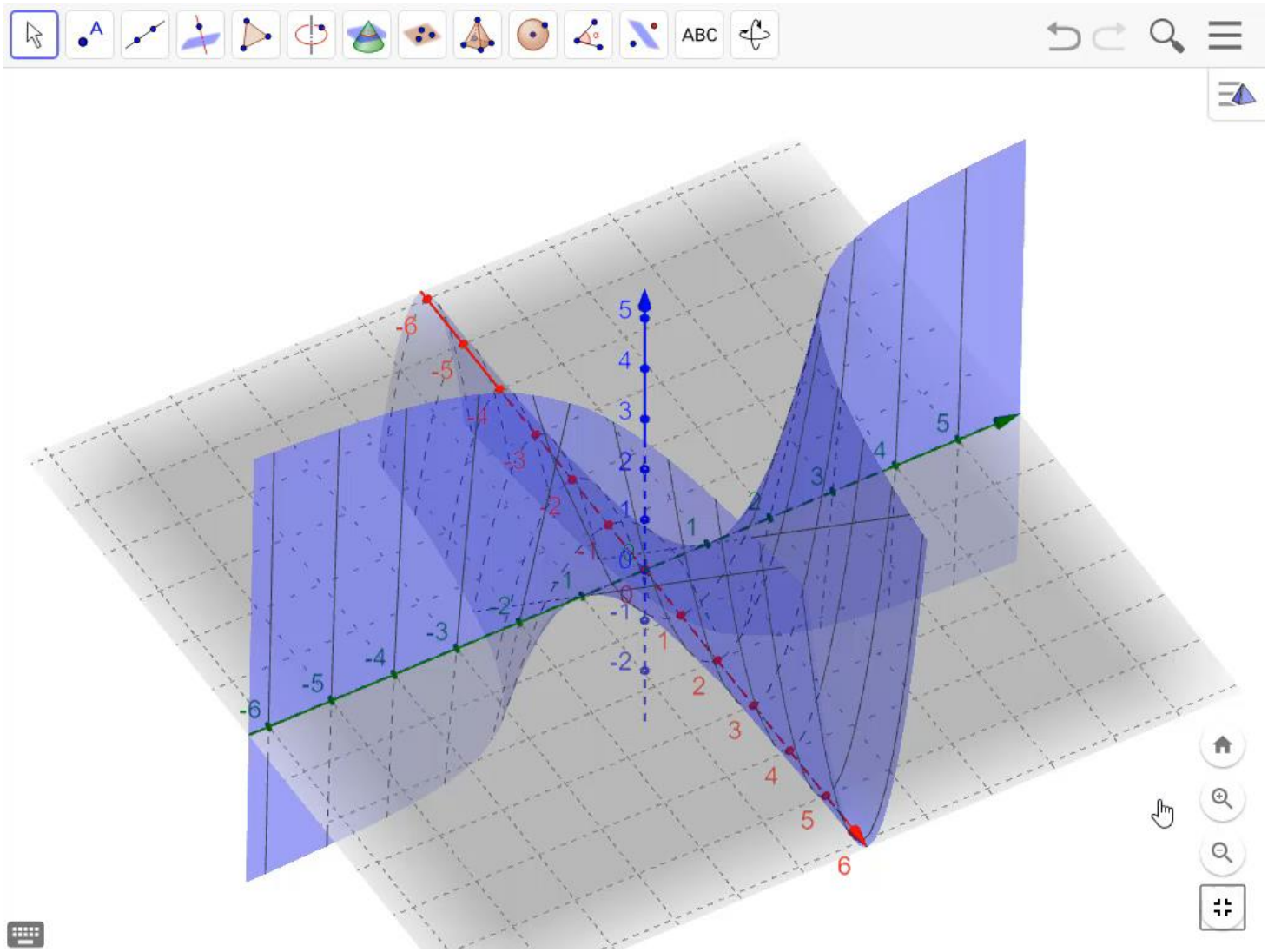
Where $R = \{(x,y) \mid 1 < x < 2, \quad 0 < y < 1\}$

Joint Prob. Density Fun. (3/19)

$$f(x,y) = xy^2$$



Joint Prob. Density Fun. (4/19)



Joint Prob. Density Fun. (5/19)

Double Integration:

$$\int_0^1 \int_1^2 xy^2 \, dx dy$$

$$\int_1^2 xy^2 \, dx = \frac{x^2}{2} y^2 \Big|_{x=1}^{x=2}$$

Joint Prob. Density Fun. (6/19)

Double Integration:

$$\int_0^1 \int_1^2 xy^2 \, dx dy$$

$$\int_1^2 xy^2 \, dx = \frac{x^2}{2} y^2 \Big|_{x=1}^{x=2} = \left(\frac{4}{2} y^2\right) - \left(\frac{1}{2} y^2\right) = \frac{3}{2} y^2$$

Joint Prob. Density Fun. (7/19)

Double Integration:

$$\int_0^1 \frac{3}{2} y^2 dy$$

$$\int_0^1 \frac{3}{2} y^2 dy = \frac{y^3}{2}\Bigg|_{y=0}^{y=1} = \left(\frac{1}{2}\right) - (0) = \frac{1}{2}$$

Joint Prob. Density Fun. (8/19)

Double Integration:

$$\therefore \int_0^1 \int_1^2 xy^2 \, dxdy = \frac{1}{2}$$

Joint Prob. Density Fun. (9/19)

Example1:

Let X and Y are two continuous random variables with a joint probability distribution $f_{XY}(x,y)$, where

$$f_{XY}(x,y) = \begin{cases} \frac{2}{5}(2x+3y), & 0 \le x \le 1, 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a) Verify that $f_{XY}(x,y)$ is a joint density function.

b) Find $P[(X,Y) \in A]$,

where $A = \{(x,y) \mid 0 < x < \frac{1}{2}, \ \frac{1}{4} < y < \frac{1}{2}\}$

Joint Prob. Density Fun. (10/19)

Example1 – Answer (1/5):

$$\int_0^1 \int_0^1 \frac{2}{5}(2x+3y)\ dxdy$$

$$\int_0^1 \left[\frac{2}{5}(x^2+3xy)\right]_{x=0}^{x=1} dy$$

$$\int_0^1 \frac{2}{5}[(1+3y)-(0)]\,dy = \int_0^1 \frac{2}{5}+\frac{6y}{5}\ dy = \left[\frac{2}{5}y+\frac{3y^2}{5}\right]_{y=0}^{y=1}$$

Joint Prob. Density Fun. (10/19)

Example1 – Answer (2/5):

$$\int_0^1 \int_0^1 \frac{2}{5}(2x+3y)\ dxdy$$

$$\left[\frac{2}{5}y + \frac{3y^2}{5}\right]_{y=0}^{y=1} = \left(\frac{2}{5}+\frac{3}{5}\right) - (0) = \frac{5}{5} = 1$$

$$\therefore \int_0^1 \int_0^1 f_{XY}(x,y)\ dxdy = 1$$

is a joint density function

Joint Prob. Density Fun. (10/19)

Example1 – Answer (3/5):

Find $P[(X,Y) \in A]$,

$$\text{where } A = \{(x,y) \mid 0 < x < \frac{1}{2},\ \frac{1}{4} < y < \frac{1}{2}\}$$

$$\int_{1/4}^{1/2} \int_{0}^{1/2} \frac{2}{5}(2x + 3y)\ dxdy$$

$$\int_{1/4}^{1/2} \left[\frac{2}{5}(x^2 + 3xy)\right]_{x=0}^{x=1/2} dy$$

Joint Prob. Density Fun. (10/19)

Example1 – Answer (4/5):

$$\int_{1/4}^{1/2}\int_{0}^{1/2}\frac{2}{5}(2x+3y)\;dxdy$$

$$\int_{1/4}^{1/2}\left[\frac{2}{5}(x^2+3xy)\right]_{x=0}^{x=1/2}dy=\int_{1/4}^{1/2}\frac{2}{5}\left(\frac{1}{4}+\frac{3}{2}y\right)-0\;dy$$

$$\int_{1/4}^{1/2}\left(\frac{1}{10}+\frac{3}{5}y\right)dy=\left[\frac{1}{10}y+\frac{3}{10}y^2\right]_{y=1/4}^{y=1/2}$$

Joint Prob. Density Fun. (10/19)

Example1 – Answer (5/5):

$$\int_{1/4}^{1/2}\int_{0}^{1/2}\frac{2}{5}(2x+3y)\ dxdy$$

$$\int_{1/4}^{1/2}\left(\frac{1}{10}+\frac{3}{5}y\right)dy=\left[\frac{1}{10}y+\frac{3}{10}y^2\right]_{y=1/4}^{y=1/2}$$

$$=\left(\frac{1}{20}+\frac{3}{40}\right)-\left(\frac{1}{40}+\frac{3}{160}\right)=0.125-0.04375=0.08125$$

Joint Prob. Density Fun. (11/19)

Example2:

Determine the value of $\boldsymbol{c}$ that makes the function $f_{XY}(x,y) = \boldsymbol{c}(x+y)$ a joint probability density function over the range $0 < x < 3$ and $x < y < x + 2$.

Joint Prob. Density Fun. (12/19)

Example2 – Answer (1/4):

Determine the value of $\boldsymbol{c}$ that makes the function $f_{XY}(x,y) = \boldsymbol{c}(x+y)$ a joint probability density function over the range $0 < x < 3$ and $x < y < x + 2$.

$$\int_0^3 \int_x^{x+2} \boldsymbol{c}(x+y)\ \ dydx = 1$$

Joint Prob. Density Fun. (12/19)

Example2 – Answer (2/4):

$$\int_0^3 \int_x^{x+2} \boldsymbol{c}(x+y)\ \ dydx = 1$$

$$\int_0^3 \left[\boldsymbol{c}\left(xy + \frac{y^2}{2}\right)\right]_{y=x}^{y=x+2} dx$$

$$\int_0^3 \boldsymbol{c}\left[\left(x^2 + 2x + \frac{(x+2)^2}{2}\right) - \left(x^2 + \frac{x^2}{2}\right)\right] dx$$

Joint Prob. Density Fun. (12/19)

Example2 – Answer (3/4):

$$\int_0^3 \int_x^{x+2} \boldsymbol{c}(x+y)\ \ dydx = 1$$

$$\int_0^3 \boldsymbol{c}\left[\left(x^2 + 2x + \frac{(x+2)^2}{2}\right) - \left(x^2 + \frac{x^2}{2}\right)\right] dx$$

$$\int_0^3 \boldsymbol{c}(4x+2)\, dx = [\boldsymbol{c}(2x^2+2x)]_{x=0}^{x=3}$$

Joint Prob. Density Fun. (12/19)

Example2 – Answer (4/4):

$$\int_0^3 \int_x^{x+2} \boldsymbol{c}(x+y)\ \ dydx = 1$$

$$[\boldsymbol{c}(2x^2+2x)]_{x=0}^{x=3} = \boldsymbol{c}[(18+6)-(0)] = 24\,\boldsymbol{c}$$

$$24\,\boldsymbol{c} = 1 \qquad \therefore \boldsymbol{c} = \frac{1}{24}$$

Joint Prob. Density Fun. (13/19)

Marginal Probability Distributions (Continuous):

The marginal distributions of the random variable X alone is:

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x, y)\ dy$$

The marginal distributions of the random variable Y alone is:

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x, y)\ dx$$

Joint Prob. Density Fun. (14/19)

Example3:

Let X and Y are two continuous random variables with a joint probability distribution $f_{XY}(x,y)$, where

$$f_{XY}(x,y) = \begin{cases} \frac{2}{5}(2x+3y), & 0 \le x \le 1, 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a) Find $f_X(x)$

b) Find $f_Y(y)$

Joint Prob. Density Fun. (15/19)

Example3 – Answer (1/2):

Find $f_X(x)$

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\ dy = \int_0^1 \frac{2}{5}(2x+3y)\ dy$$

$$\int_0^1 \frac{2}{5}(2x+3y)\ dy = \left[\frac{2}{5}\left(2xy + \frac{3y^2}{2}\right)\right]_{y=0}^{y=1} = \left(\frac{4x}{5} + \frac{3}{5}\right) - (0)$$

$$f_X(x) = \frac{4x}{5} + \frac{3}{5}$$

Joint Prob. Density Fun. (15/19)

Example3 – Answer (2/2):

Find $f_Y(y)$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\ dx = \int_0^1 \frac{2}{5}(2x+3y)\ dx$$

$$\int_0^1 \frac{2}{5}(2x+3y)\ dx = \left[\frac{2}{5}(x^2+3xy)\right]_{x=0}^{x=1} = \left(\frac{2}{5}+\frac{6y}{5}\right) - (0)$$

$$f_Y(y) = \frac{2}{5} + \frac{6y}{5}$$

Joint Prob. Density Fun. (16/19)

Mean from a Joint Distribution (Continuous):

Mean for the random variable X alone is:

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\ dx = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f_{XY}(x,y)\ dydx$$

Mean for the random variable Y alone is:

$$E(Y) = \int_{-\infty}^{\infty} y f_Y(y)\ dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y f_{XY}(x,y)\ dydx$$

Joint Prob. Density Fun. (17/19)

Variance from a Joint Distribution (Continuous):

Variance for the random variable X alone is:

$$V(X) = E(X^2) - \big(E(X)\big)^2$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\ dx$$

and s.d. of $X = \sigma_X = \sqrt{V(X)}$

Variance for the random variable Y alone is:

$$V(Y) = E(Y^2) - \big(E(Y)\big)^2$$

$$E(Y^2) = \int_{-\infty}^{\infty} y^2 f_Y(y)\ dy$$

and s.d. of $Y = \sigma_Y = \sqrt{V(Y)}$

Joint Prob. Density Fun. (18/19)

Example4:

Let X and Y are two continuous random variables with a joint probability distribution $f_{XY}(x,y)$, where

$$f_{XY}(x,y) = \begin{cases} \frac{2}{5}(2x+3y), & 0 \le x \le 1, 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a) Find $E(X)$

b) Find $E(Y)$

c) Find σ_X

Joint Prob. Density Fun. (19/19)

Example4 – Answer (1/4):

$$f_X(x) = \frac{4x}{5} + \frac{3}{5}$$

Find $E(X)$

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\ dx = \int_0^1 \frac{4x^2}{5} + \frac{3x}{5}\ dx$$

$$\int_0^1 \frac{4x^2}{5} + \frac{3x}{5}\ dx = \left[\frac{4x^3}{15} + \frac{3x^2}{10}\right]_{x=0}^{x=1} = \left(\frac{4}{15} + \frac{3}{10}\right) - (0)$$

$$E(X) = \frac{4}{15} + \frac{3}{10} = 0.5667$$

Joint Prob. Density Fun. (19/19)

Example4 – Answer (2/4):

$$f_Y(y) = \frac{2}{5} + \frac{6y}{5}$$

Find $E(Y)$

$$E(Y) = \int_{-\infty}^{\infty} y f_Y(y)\ dy = \int_0^1 \frac{2y}{5} + \frac{6y^2}{5}\ dy$$

$$\int_0^1 \frac{2y}{5} + \frac{6y^2}{5}\ dy = \left[\frac{y^2}{5} + \frac{2y^3}{5}\right]_{y=0}^{y=1} = \left(\frac{1}{5} + \frac{2}{5}\right) - (0)$$

$$E(Y) = \frac{1}{5} + \frac{2}{5} = \frac{3}{5} = 0.6$$

Joint Prob. Density Fun. (19/19)

Example4 – Answer (3/4):

$E(X) = 0.5667$

Find σ_X

$$\sigma_X = \sqrt{V(X)} = \sqrt{E(X^2) - \big(E(X)\big)^2}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\ dx = \int_0^1 \frac{4x^3}{5} + \frac{3x^2}{5}\ dx$$

$$E(X^2) = \left[\frac{x^4}{5} + \frac{x^3}{5}\right]_{x=0}^{x=1} = \left(\frac{1}{5} + \frac{1}{5}\right) - (0) = \frac{2}{5} = 0.4$$

Joint Prob. Density Fun. (19/19)

Example4 – Answer (4/4):

$E(X) = 0.5667$

$E(X^2) = 0.4$

Find σ_X

$$\sigma_X = \sqrt{V(X)} = \sqrt{E(X^2) - \big(E(X)\big)^2}$$

$$\sigma_X = \sqrt{V(X)} = \sqrt{0.4 - (0.5667)^2} = \sqrt{0.07885111}$$

$$\sigma_X = 0.2808044$$

Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-

Lecture #8: https://www.youtube.com/watch?v=APvrmntE_hA&list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-&index=9

Notes Lec 5 – 8: https://www.youtube.com/watch?v=BVXwtICE_no&list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-&index=10

Thank You

Dr. Ahmed Hagag
ahagag@fci.bu.edu.eg